Active Control of Vehicle Attitude with Roll Dynamics ⋆

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Abstract: In this work an integrated attitude control of a vehicle is designed. The actuators considered are the active front steering, the rear torque vectoring, and the semi–active suspensions. We design also an algorithm for the saturation management. The resulting controller is hybrid. In nominal conditions, when saturation conditions do not occur, a feedback guarantees exponential tracking of the reference trajectories. In critical conditions, a hybrid feedback law assigns higher priority to some states to improve the tracking. It is shown that the saturation management improves the controller performance. Some simulations results are provided, showing the performance of the proposed controller.

1. INTRODUCTION

In the latest years, the development of an integrated philosophy has changed the way researchers and engineers design feedback devices in automotive control. With respect to some years ago, a much larger computational power has been made available to the control unit, due to the improvement of the electronics and to the increasing number of available customer features and technologies. This allows designers of automotive control systems to cope with many kinds of requests and constraints. One of the more interesting and appealing aspects is the integration of the various vehicle subsystems. This integration is generally regarded as a full coordination of all basic control functions, since presently a fully integrated control, in substitution of standardized controls, seems not feasible.

The design of active attitude control systems, addressed in this paper, is one of the main research topics in vehicle control area. These devices modify the vehicle dynamics imposing forces or moments to the vehicle body in different ways (see, e.g. Burgio [2006], Karbalaei [2007], Baslamisli [2007], Ackermann [1995], Malan [1994]), and can now make use of smart sensors (for example, the so–called intelligent tires, Ergen [2009]), allowing precise and distributed measurements from the environment, to increase the performance of the control action, the vehicle stability, and the safety and comfort of the driver. On top of that, hierarchical and hybrid structures guarantee increased performance and robustness of control strategies, taking into account the interactions among vehicle, driver and environment, considered in parallel in one core algorithm.

An important design factor to be considered in the standalone or integrated controller design is the actuator saturation, which limits the maximum obtainable performance. In an integrated control structure more power is available for control, thus potentially limiting the saturation occurrences. In all cases, it is important to manage critical situations, whenever actuators are not physically able to apply the required input.

In some previous works (Borri [2007], Bianchi [2008], Bianchi [2010]), we have addressed vehicle attitude control by using active front steering and rear torque vectoring. The application of an adaptive feedback linearization control (Sastry [1989]) has been proposed to improve stability in the presence of deviations of the vehicle parameters from the nominal values, and of rapid variations of road conditions. In that works, roll motion has been neglected and no countermeasures for actuator saturations have been considered. Nevertheless, considering the roll dynamics can improve the performance of the controller, as well as the management of saturations occurring in the actuators.

As far as roll control is concerned, a large number of technologies regarding electronically controlled active and semi–active suspension systems have been developed in the last twenty years, oriented both on comfort and handling improvement (see e.g. Savaresi [2009], Canale [2006]). Another research line has tackled the problem of control of linear and nonlinear systems with input constraints and saturations (e.g. see Lin [1996], Yang [1993]).

In this work, we consider semi–active suspensions for the roll control, and a hybrid control law is designed for the saturation management. Since an actuator saturation can be regarded as a loss of a degree of freedom in the control action, resulting in the impossibility of fulfilling the original goals, the controller detects this condition and determines the new requirements to be met, on the basis of predetermined priorities.

The contribution of the present work is twofold. First, we propose a fully–integrated controller, achieving the tracking of yaw rate, lateral velocity and roll dynamics (angle and velocity), by means of three actuators, in nominal conditions. Second, we propose a methodology, based on a hybrid controller, for the management of the actuators in saturation conditions, based on

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fixed priorities on the states to be tracked, according to their importance for the vehicle attitude.

The paper is organized as follows. In Section 2, the mathematical model of the vehicle is presented, and the control problem is stated. In Section 3, the nonlinear technique is developed and the saturation management is described. In Section 4, the proposed controller is tested with simulations and comparisons. Some comments conclude the paper.

2. MATHEMATICAL MODEL AND PROBLEM FORMULATION

For simplicity, we consider the model of a rear–wheel driven vehicle. The actuators considered in this work are

1. Active Front Steer (AFS), which imposes an incremental steer angle on top of the driver's input. The control is then actuated through the front axle tire characteristic.
2. Rear Torque Vectoring (RTV), which distributes the torque in the rear axle, usually to improve vehicle traction, handling and stability. The control is then actuated through the rear axle tire characteristics.
3. Semi–active suspensions (SAS), which are able to change the damping coefficient of the shock absorber in a continuous interval, differently from passive systems.

The mathematical model is derived under the following assumptions, which are verified in a large number of situations and which mitigate the complexity of the vehicle dynamics

- The vehicle moves on a horizontal plane;
- The longitudinal velocity is constant, so that vehicle shaking/pitch motions can be neglected;
- The steering system is rigid, so that the angular position of the front wheels is uniquely determined by the steering wheel position;
- The wheels masses are much lower than the vehicle one, so the steering action does not affect the position of the centre of mass of the entire vehicle;
- The vehicle takes large radius bends and the road wheel angles are "small" (less than $10°$), i.e. the bend curve radius is much higher than the vehicle width;
- The aerodynamic resistance and the wind lateral thrust are not considered;
- The tire vertical loads are constant;
- The actuators are ideally modelled.

As a consequence of the previous assumptions, the vehicle model has four degrees of freedom: lateral velocity, yaw rate, roll angle and velocity. Following Guiggiani [2007], one can consider the lateral acceleration of the vehicle

$$m(\dot{v}_y + v_x\omega_z) = \mu(F_{y,f} + F_{y,r}) + m_s h_d \dot{\omega}_x \quad (1)$$

where m is the vehicle mass, v_y, ω_z, ω_x are the lateral, yaw, and roll velocities, $h_d = h - d$, h is the center of gravity height, d is the roll center height, m_s is the sprung mass, μ is the road–tire friction coefficient, and $F_{y,f}$, $F_{y,r}$ the lateral (front/rear) tire forces. The lateral forces $F_{y,f} = F_{y,f}(\alpha_f)$, $F_{y,r} = F_{y,r}(\alpha_r)$ depend on the slip angles

$$\alpha_f = \delta_c + \alpha_{f,0} = \delta_c + \delta_d + \gamma_f\alpha_x - \frac{v_y + l_f\omega_z}{v_x}$$
$$\alpha_r = \gamma_r\alpha_x - \frac{v_y - l_r\omega_z}{v_x}$$

where δ_c, δ_d are the control, driver components of the wheel angle, γ_f, γ_r are front/rear sensitivities with respect to the roll angle α_x, and l_f, l_r are the distances from the center of gravity to the front/rear axle. A simple but accurate representation of the lateral force functions is given by (Pacejka [2005])

$$\begin{aligned} F_{y,f}(\alpha_f) &= C_{y,f}\sin(A_{y,f}\arctan(B_{y,f}\alpha_f)) \\ F_{y,r}(\alpha_r) &= C_{y,r}\sin(A_{y,r}\arctan(B_{y,r}\alpha_r)) \end{aligned} \quad (2)$$

where $A_{y,f},\ A_{y,r},\ B_{y,f},\ B_{y,r},\ C_{y,f},\ C_{y,r}$ are experimental constants.

The vehicle yaw dynamics can be expressed considering the presence of RTV actuators

$$J_z\dot{\omega}_z = \mu(F_{y,f}l_f - F_{y,r}l_r) + M_z + J_{zx}\dot{\omega}_x \quad (3)$$

where J_z is the vehicle inertia momentum about the z axis, J_{zx} is the product of inertia about the axes z, x, and M_z is RTV yaw moment.

The vehicle roll angular acceleration can be expressed as

$$\begin{aligned} \dot{\alpha}_x &= \omega_x \\ J_r\dot{\omega}_x &= -b_x\omega_x - (k_x - m_s g h_d)\alpha_x + J_{zx}\dot{\omega}_z \\ &\quad + m_s h_d(\dot{v}_y + v_x\omega_z) \end{aligned} \quad (4)$$

where J_x is the vehicle inertia momentum about the x axis, $J_r = J_x + m_s h_d^2$, b_x is the suspension roll damping, k_x is the suspension roll stiffness, and g is the gravity acceleration constant.

From (1), (3), (4) we get the mathematical model of a vehicle with yaw, lateral and roll dynamics

$$\begin{aligned} \dot{\omega}_z &= \frac{1}{J_{z,e}}\Big[\mu(F_{y,f}l_f - F_{y,r}l_r) + M_z\Big] \\ &\quad + k_m k_z h_e \mu(F_{y,f} + F_{y,r}) \\ &\quad - k_m k_z\Big[b_x\omega_x + (k_x - m_s g h_d)\alpha_x\Big] \\ \dot{v}_y &= -\omega_z v_x + \frac{1}{m_e}\mu(F_{y,f} + F_{y,r}) \\ &\quad + k_m k_z h_e\Big[\mu(F_{y,f}l_f - F_{y,r}l_r) + M_z\Big] \\ &\quad - k_m h_e\Big[b_x\omega_x + (k_x - m_s g h_d)\alpha_x\Big] \\ \dot{\omega}_x &= -k_m\Big[b_x\omega_x + (k_x - m_s g h_d)\alpha_x\Big] \\ &\quad + k_m k_z\Big[\mu(F_{y,f}l_f - F_{y,r}l_r) + M_z\Big] \\ &\quad + k_m h_e \mu(F_{y,f} + F_{y,r}) \\ \dot{\alpha}_x &= \omega_x \end{aligned} \quad (5)$$

where $k_m = 1/J_{x,e}$ and

$$
\begin{aligned}
J_{x,e} &= J_r - \frac{J_{zx}^2}{J_z} - \frac{m_s^2 h_d^2}{m} \\
m_e &= \frac{m}{1 + m_s k_m h_d h_e}, \qquad J_{z,e} = \frac{J_z}{1 + k_m k_z J_{zx}} \\
h_e &= \frac{m_s}{m} h_d, \qquad\qquad k_z = \frac{J_{zx}}{J_z}.
\end{aligned}
$$

As anticipated, we consider v_x constant. Moreover, the control inputs that we consider are M_z, and the differences

$$
\Delta F_{y,f} = F_{y,f} - F_{y,f,0}, \qquad F_{y,f,0} := F_{y,f}(\alpha_{f,0})
$$

$$
\Delta b_x = b_x - b_{x,0}
$$

with $b_{x,0}$ the damping when the SAS system is not active. Clearly, the real active front input is the control angle δ_c which can be determined by inverting (2), obtaining

$$
\delta_c = \begin{cases} -\delta_d + \frac{v_y + l_f \omega_z}{v_x} + F_{y,f}^{-1}(\bar{F}) & \text{if } |\bar{F}| \leq F_{y,f}(\alpha_{f,\text{sat}}) \\ -\delta_d + \frac{v_y + l_f \omega_z}{v_x} \pm \alpha_{f,\text{sat}} & \text{otherwise} \end{cases}
$$

with $\bar{F}$ a fixed value to be imposed by the AFS.

The control aim is to track asymptotically some bounded references, with bounded derivatives, for ω_z, v_y, α_x and ω_x. More precisely, the reference generator is

$$
\begin{aligned}
\dot{\omega}_{z,\text{ref}} &= \frac{1}{J_{z,\text{ref}}} \Big[\mu_{\text{ref}} (F_{y,f,\text{ref}} l_f - F_{y,r,\text{ref}} l_r) \Big] \\
&\quad + k_m k_z h_e \mu_{\text{ref}} (F_{y,f,\text{ref}} + F_{y,r,\text{ref}}) \\
&\quad - k_m k_z \Big[b_{x,\text{ref}} \omega_{x,\text{ref}} + (k_{x,\text{ref}} - m_s g h_d) \alpha_{x,\text{ref}} \Big] \\
\dot{v}_{y,\text{ref}} &= -\omega_{z,\text{ref}} v_x + \frac{1}{m_e} \mu_{\text{ref}} (F_{y,f,\text{ref}} + F_{y,r,\text{ref}}) \\
&\quad + k_m k_z h_e \Big[\mu_{\text{ref}} (F_{y,f,\text{ref}} l_f - F_{y,r,\text{ref}} l_r) \Big] \\
&\quad - k_m h_e \Big[b_{x,\text{ref}} \omega_{x,\text{ref}} + (k_{x,\text{ref}} - m_s g h_d) \alpha_{x,\text{ref}} \Big] \\
\dot{\omega}_{x,\text{ref}} &= -k_m \Big(b_{x,\text{ref}} \omega_{x,\text{ref}} + k_{x,\text{ref}} \alpha_{x,\text{ref}} \Big) \\
&\quad + k_m k_z \Big[\mu_{\text{ref}} (F_{y,f,\text{ref}} l_f - F_{y,r,\text{ref}} l_r) \Big] \\
&\quad + k_m h_e \mu_{\text{ref}} (F_{y,f,\text{ref}} + F_{y,r,\text{ref}}) \\
\dot{\alpha}_{x,\text{ref}} &= \omega_{x,\text{ref}}
\end{aligned}
\tag{6}
$$

where $J_{z,\text{ref}}$, μ_{ref}, $b_{x,\text{ref}}$, $k_{x,\text{ref}}$ are appropriate parameters and $F_{y,f,\text{ref}}$, $F_{y,r,\text{ref}}$ are ideal curves, depending on

$$
\alpha_{f,\text{ref}} = \delta_d + \gamma_f \alpha_{x,\text{ref}} - \frac{v_{y,\text{ref}} + l_f \omega_{z,\text{ref}}}{v_x}
$$

$$
\alpha_{r,\text{ref}} = \gamma_r \alpha_{x,\text{ref}} - \frac{v_{y,\text{ref}} - l_r \omega_{z,\text{ref}}}{v_x}.
$$

In particular, we set

$$
\begin{aligned}
&F_{y,j,\text{ref}}(\alpha_{j,\text{ref}}) = \\
&= \begin{cases} F_{y,j}(-\bar{\alpha}_{j,\text{ref}}) + F'_{y,j}(\bar{\alpha}_{j,\text{ref}})(\alpha_{j,\text{ref}} + \bar{\alpha}_{j,\text{ref}}) & \alpha_{j,\text{ref}} < -\bar{\alpha}_{j,\text{ref}} \\ F_{y,j}(\alpha_{j,\text{ref}}) & |\alpha_{j,\text{ref}}| \leq \bar{\alpha}_{j,\text{ref}} \\ F_{y,j}(\bar{\alpha}_{j,\text{ref}}) + F'_{y,j}(\bar{\alpha}_{j,\text{ref}})(\alpha_{j,\text{ref}} - \bar{\alpha}_{j,\text{ref}}) & \alpha_{j,\text{ref}} > \bar{\alpha}_{j,\text{ref}} \end{cases}
\end{aligned}
$$

with $j = f, r$, where $\bar{\alpha}_{j,\text{ref}}$ is a limit value for the slip angle, above which the control action is needed.

3. DESIGN OF A STATE–FEEDBACK LINEARIZING CONTROL LAW WITH SATURATION MANAGEMENT

In this section a state–feedback linearizing control, imposing desired behaviors for yaw, lateral and roll dynamics, is designed. Then a saturation management algorithm is proposed.

3.1 State–Feedback Linearizing Control Law

Let us impose

$$
\begin{aligned}
\dot{\omega}_z &= \nu_1 := \dot{\omega}_{z,\text{ref}} - k_1(\omega_z - \omega_{z,\text{ref}}) \\
\dot{v}_y &= \nu_2 := \dot{v}_{y,\text{ref}} - k_2(v_y - v_{y,\text{ref}}) \\
\dot{\omega}_x &= \nu_3 := \dot{\omega}_{x,\text{ref}} - k_3(\omega_x - \omega_{x,\text{ref}}) - k_4(\alpha_x - \alpha_{x,\text{ref}})
\end{aligned}
\tag{7}
$$

$k_i > 0$, $i = 1, \cdots, 3$, $k_4 \geq 0$, where the term $k_4(\alpha_x - \alpha_{x,\text{ref}})$ is an integral action for the tracking of $\omega_{x,\text{ref}}$. To fulfill (7), the following control law has to be imposed

$$
\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} \Delta F_{y,f} \\ M_z \\ \Delta b_x \end{pmatrix} = \mathcal{C}^{-1} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix}
\tag{8}
$$

where

$$
\mathcal{C} = \begin{pmatrix} \mu \Big(k_m k_z h_e + \frac{l_f}{J_{z,e}} \Big) & \frac{1}{J_{z,e}} & -k_m k_z \omega_x \\ \mu \Big(\frac{1}{m_e} + k_m k_z h_e l_f \Big) & k_m k_z h_e & -k_m h_e \omega_x \\ \mu k_m (h_e + k_z l_f) & k_m k_z & -k_m \omega_x \end{pmatrix}
\tag{9}
$$

and

$$
\begin{aligned}
r_1 &= \nu_1 - k_m k_z h_e \mathcal{F}_y - \frac{1}{J_{z,e}} \mathcal{T}_y + k_m k_z \mathcal{D}_y \\
r_2 &= \nu_2 + \omega_z v_x - \frac{1}{m_e} \mathcal{F}_y - k_m k_z h_e \mathcal{T}_y + k_m h_e \mathcal{D}_y \\
r_3 &= \nu_3 - k_m h_e \mathcal{F}_y - k_m k_z \mathcal{T}_y + k_m \mathcal{D}_y \\
\mathcal{F}_y &= \mu (F_{y,f,0} + F_{y,r}) \\
\mathcal{T}_y &= \mu (F_{y,f,0} l_f - F_{y,r} l_r) \\
\mathcal{D}_y &= b_{x,0} \omega_x + (k_x - m_s g h_d) \alpha_x.
\end{aligned}
$$

In the following we assume that

$$
k_m k_z^2 J_{z,e} \neq 1, \qquad m_e h_e^2 k_m \neq 1
$$

which is reasonable since this is verified for an appropriate choice of the vehicle parameters. Therefore, since the first column of $\mathcal{C}$ depends on μ and the third on ω_x, $\mathcal{C}$ is invertible if and only if $\mu \neq 0$ and $\omega_x \neq 0$. The inverse of (9) is given by

$$
\mathcal{C}^{-1} = \begin{pmatrix} 0 & \frac{1}{\mu} \frac{m_e}{1 - m_e h_e^2 k_m} & -\frac{1}{\mu} \frac{m_e h_e}{1 - m_e h_e^2 k_m} \\ \frac{J_{z,e}}{1 - k_m k_z^2 J_{z,e}} & -\frac{m_e l_f}{1 - m_e h_e^2 k_m} & \frac{m_e h_e l_f}{1 - m_e h_e^2 k_m} - \frac{k_z J_{z,e}}{1 - k_m k_z^2 J_{z,e}} \\ \frac{1}{\omega_x} \frac{k_z J_{z,e}}{1 - k_m k_z^2 J_{z,e}} & \frac{1}{\omega_x} \frac{m_e h_e}{1 - m_e h_e^2 k_m} & \frac{1}{\omega_x} \frac{-(1 - m_e h_e^2 k_m^2 k_z^2 J_{z,e})}{k_m (1 - m_e h_e^2 k_m)(1 - k_m k_z^2 J_{z,e})} \end{pmatrix}.
$$

Therefore, it is clear that when $\mu = 0$ and/or $\omega_x = 0$, the control law (8) is not computable. Physically, when $\mu \to 0$ and/or $\omega_x \to 0$, some actuators may saturate.

3.2 Saturation Management

In the following we propose a hybrid approach to the design of a procedure to manage saturation in the actuators. With $j_a = 1$ we denote the AFS, with $j_a = 2$ the RTV, and with $j_a = 3$ the SAS. If the j_a^{th} actuator saturates, $j_a = 1, 2, 3$, the corresponding control is $u_{j_a} = \pm u_{j_a,\max}$. Considering the requirements expressed by (7), we assign higher priority to the fulfillment of those for ω_z, and lower priority to those for ω_x. The actuators that do not saturate will ensure the fulfillment of the requirements with higher priority. To show the procedure in general, let

$$\dot{x}_i = \nu_i, \qquad i = 1, \cdots, m \tag{10}$$

be the set of requirements, generalizing (7), ordered according to the priority (first = higher priority, last = lower priority), and let

$$\mathcal{C}u = \mathcal{C}\begin{pmatrix} u_1 \\ \vdots \\ u_m \end{pmatrix} = \begin{pmatrix} r_1 \\ \vdots \\ r_m \end{pmatrix} = R \tag{11}$$

be the general equation for determining the control law satisfying (10). We partition u into a vector of non–saturating and saturating inputs

$$u = \begin{pmatrix} u_{ns} \\ u_s \end{pmatrix}, \quad u_{ns} \in \mathbb{R}^{m_{ns}}, \quad u_s \in \mathbb{R}^{m_s}, \quad m_{ns} + m_s = m.$$

We operate the corresponding partition on $\mathcal{C}$, so that (11) becomes

$$\begin{pmatrix} \mathcal{C}_{ns} & \mathcal{C}_s \end{pmatrix} \begin{pmatrix} u_{ns} \\ u_s \end{pmatrix} = R.$$

Finally, to select the first m_{ns} requirements

$$\begin{pmatrix} \bar{\mathcal{C}}_{ns} & \bar{\mathcal{C}}_s \end{pmatrix} = \begin{pmatrix} I_{m_{ns} \times m_{ns}} & 0_{m_{ns} \times m_s} \end{pmatrix} \begin{pmatrix} \mathcal{C}_{ns} & \mathcal{C}_s \end{pmatrix}$$

$$\bar{R} = \begin{pmatrix} I_{m_{ns} \times m_{ns}} & 0_{m_{ns} \times m_s} \end{pmatrix} R$$

so that one calculates the non–saturating controls

$$u_{ns} = \bar{\mathcal{C}}_{ns}^{-1}\Big(\bar{R} - \bar{\mathcal{C}}_s u_s\Big).$$

To illustrate the proposed saturation management, let us consider some examples in the case under study. Let us assume that the SAS actuator, calculated according to (8), saturates. Hence,

$$\begin{pmatrix} \Delta F_{y,f} \\ M_z \end{pmatrix} = \begin{pmatrix} \mu\Big(k_m k_z h_e + \frac{l_f}{J_{z,e}}\Big) & \frac{1}{J_{z,e}} \\ \mu\Big(\frac{1}{m_e} + k_m k_z h_e l_f\Big) & k_m k_z h_e \end{pmatrix}^{-1} \times$$

$$\times \left(\begin{pmatrix} r_1 \\ r_2 \end{pmatrix} \pm \Delta b_{x,\max} \begin{pmatrix} k_m k_z \omega_x \\ k_m h_e \omega_x \end{pmatrix} \right).$$

Note that if the saturation is due to the fact that $\omega_x = 0$, this control law reduces

$$\begin{pmatrix} \Delta F_{y,f} \\ M_z \end{pmatrix} = \begin{pmatrix} \mu\Big(k_m k_z h_e + \frac{l_f}{J_{z,e}}\Big) & \frac{1}{J_{z,e}} \\ \mu\Big(\frac{1}{m_e} + k_m k_z h_e l_f\Big) & k_m k_z h_e \end{pmatrix}^{-1} \begin{pmatrix} r_1 \\ r_2 \end{pmatrix}$$

while $\Delta b_x = \pm \Delta b_{x,\max}$.

Following the same procedure, if both SAS and AFS saturate, one has

$$M_z = J_{z,e}\bigg(r_1 \mp \Delta F_{y,f,\max}\mu\Big(k_m k_z h_e + \frac{l_f}{J_{z,e}}\Big) \pm \Delta b_{x,\max} k_m k_z \omega_x\bigg)$$

and if the saturations are due to $\mu = 0$, $\omega_x = 0$, this control law reduces to

$$\Delta F_{y,f} = \pm \Delta F_{y,f,\max}, \quad M_z = J_{z,e} r_1, \quad \Delta b_x = \pm \Delta b_{x,\max}.$$

4. SIMULATION RESULTS

In this section, we provide simulation results of the proposed control technique. We consider two simulation sets. First, we compare the proposed integrated solution with a linearizing feedback controller (Borri [2007], Bianchi [2008]) based on a lower number of actuators. Second, we show the improvement ensured by the hybrid saturation management scheme with respect to a classical fixed saturation scheme.

The parameters of the vehicle are

$m = 1550$ kg	$m_s = 150$ kg	$\mu = 1$
$l_f = 1.17$ m	$l_r = 1.43$ m	$h_d = 0.5$ m
$J_z = 2300\ \mathrm{kg \cdot m^2}$	$J_x = 350\ \mathrm{kg \cdot m^2}$	$J_{zx} = 50\ \mathrm{kg\, m^2}$
$A_{yf} = 1.81$	$B_{yf} = 7.2$	$C_{yf} = 8854$
$A_{yr} = 1.68$	$B_{yr} = 11$	$C_{yr} = 8394$
$k_x = 150{,}000\ \mathrm{Nm/rad}$	$b_{x,0} = 7{,}000\ \mathrm{Nm\, rad/s}$	
$\gamma_f = -0.05$	$\gamma_r = 0.05$	

while for the reference generation we have considered the following set of values

$J_{z,\mathrm{ref}} = J_z$	$\mu_{\mathrm{ref}} = \mu$	$\bar{\alpha}_{f,\mathrm{ref}} = 0.08$
$\bar{\alpha}_{r,\mathrm{ref}} = 0.04$	$k_{x,\mathrm{ref}} = k_x$	$b_{x,\mathrm{ref}} = 11800.$

The control inputs are restricted to the following intervals (see Spelta [2010])

$$\Delta F_{y,f} \in [-0.95 C_{yf} - F_{y,f,0}, 0.95 C_{yf} - F_{y,f,0}]$$
$$M_z \in [-10000, 10000], \qquad \Delta b_x \in [-2500, 35000].$$

4.1 Comparison with other controllers

The first test maneuver is a step steer of $120°$ with longitudinal velocity of 28 m/s (100.8 km/h). It is useful to compare the performance of the controller with the AFS, RTV, SAS actuators, defined in Section 3.1, with a state feedback linearizing control law with two actuators (AFS and RTV), as defined in Borri [2007], Bianchi [2008], designed to track lateral velocity and yaw rate. Thanks to the presence of the SAS, whose primary goal is the reduction of the oscillating behavior, the proposed controller obviously performs better results for the roll tracking, namely α_x, ω_x track more closely the references $\alpha_{x,\mathrm{ref}}, \omega_{x,\mathrm{ref}}$ (see Figure 1.c-d), as expected. Here the reference model has an increased value of the suspension roll damping (i.e. $b_{x,\mathrm{ref}} > b_{x,0}$). Moreover, the additional actuator influences the behavior of the other states and inputs. In particular, one can note an improved tracking of the lateral velocity (Figure 1.a), while the yaw rate behavior is similar in both cases (Figure 1.b), and is non–zero because of the AFS saturation (Figure 2.a). An analysis of the control inputs also shows higher values of RTV action in the presence of three actuators.

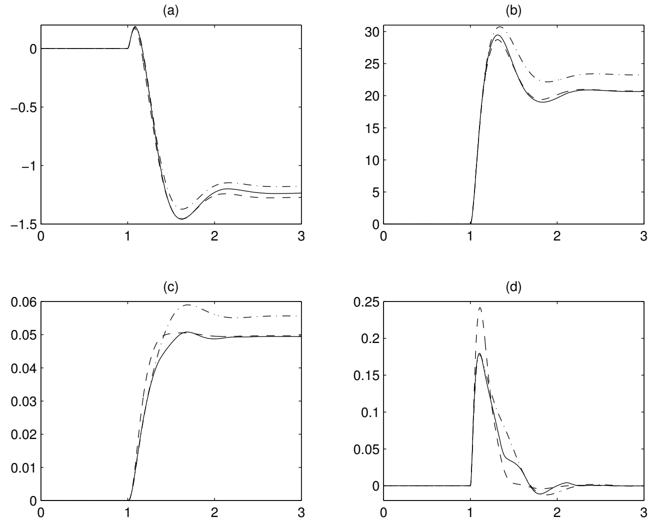

Fig. 1. (a) $v_{y,\mathrm{ref}}$ (dash–dot), v_y with 3 actuators (solid), v_y with 2 actuators (dashed) [m/s] vs time [s]; (b) $\omega_{z,\mathrm{ref}}$ (dash–dot), ω_z with 3 actuators (solid), ω_z with 2 actuators (dashed) [deg/s] vs time [s]; (c) $\alpha_{x,\mathrm{ref}}$ (dash–dot), α_x with 3 actuators (solid), α_x with 2 actuators (dashed) [deg/s] vs time [s]; (dashed) [deg/s] vs time [s]; (d) $\omega_{x,\mathrm{ref}}$ (dash–dot), ω_x with 3 actuators (solid), ω_x with 2 actuators (dashed) [m/s] vs time [s].

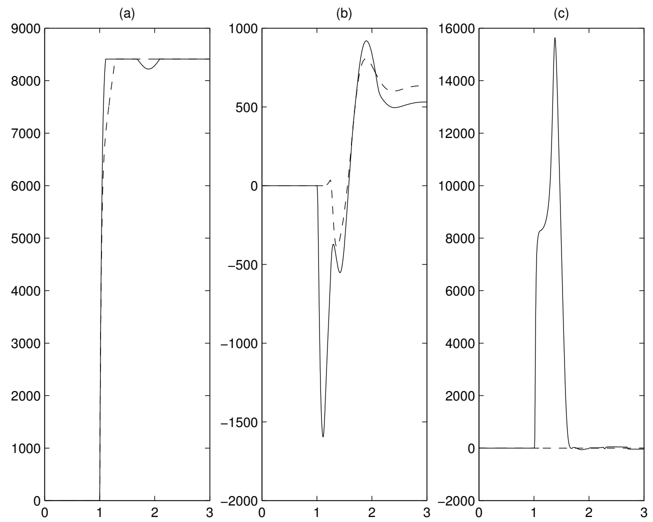

Fig. 2. (a) $F_{y,f}$ with 3 actuators (solid), $F_{y,f}$ with 2 actuators (dashed) [N] vs time [s]; (b) M_z with 3 actuators (solid), M_z with 2 actuators (dashed) [Nm] vs time [s]; (c) b_x with 3 actuators (solid), b_x with 2 actuators (dashed) [Nm·rad/s] vs time [s]

4.2 Performance of the Saturation Management

In this section we compare the proposed controller $\mathcal{C}_{sm}$, with saturation management, and a controller $\mathcal{C}_w$, in which the saturation is treated, as done usually in applications, simply by saturating the input value, without a change of priority in the control specifications.

The maneuver considered, shown in Figure 4.d, is a double step steer of 120° with longitudinal velocity of 33 m/s (118 km/h). Figure 3 show the evolution of the state variables with the two controllers. The algorithm of saturation management gives tracking priority to ω_z, as shown in Figure 3.b: while the error with $\mathcal{C}_w$ grows immediately, as consequence of the AFS saturation shown in Figure 5.a, the controller $\mathcal{C}_{sm}$ succeeds to reduce the error e_{ω_z}. This improvement is achieved thanks to the RTV compensation. Clearly, this is paid with a larger error on lateral velocity tracking (see Fig. 3.a), because of the priority scheme. However, with $\mathcal{C}_{sm}$ and with the priority on the yaw rate tracking, the handling of the vehicle has been improved, as testified by the spatial trajectory (see Fig. 6), although this is not one of the control goals. In fact, an enhanced yaw tracking improves the agility and cornering capabilities of the vehicle. The roll tracking is displayed in Figure 3.c-d, with a performance that can be considered comparable in both cases. Figure 4.a shows the total force of the front axle, Figure 4.b the yaw moment, and Figure 4.c the damping coefficient variation. The control angle δ_c due to AFS is shown in Figure 4.d, which results to be lower, in absolute value, with $\mathcal{C}_{sm}$ than with $\mathcal{C}_w$, because most of work in tracking ω_z is due to the additional RTV action. This does not mean that the front axle is not exploited at its maximum extent, since the AFS soon saturates in both cases (see Fig. 5). We also note that the SAS system saturates very soon after the AFS saturation, in both cases, meaning that this actuator is not able to compensate the lack in control action, due to the saturation of the other actuators.

In general, we can conclude that the proposed algorithm for the saturation management is able to achieve a better and deeper utilization of all the actuators in saturation conditions, resulting in an improved handling and cornering behavior.

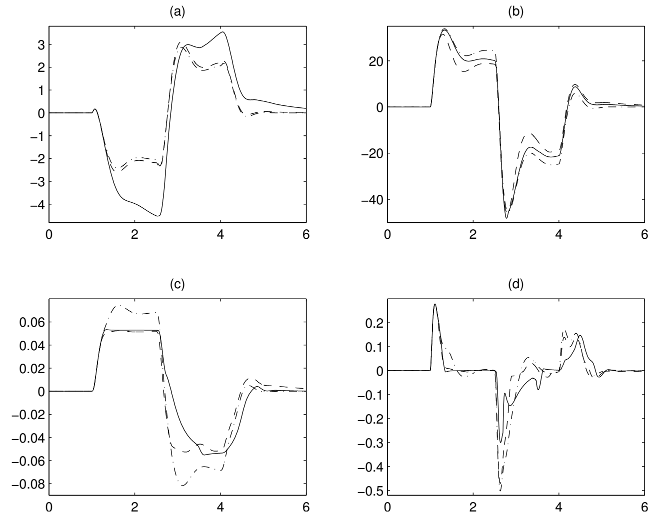

Fig. 3. (a) $v_{y,\mathrm{ref}}$ (dash–dot), v_y with 3 actuators (solid), v_y with 2 actuators (dashed) [m/s] vs time [s]; (b) $\omega_{z,\mathrm{ref}}$ (dash–dot), ω_z with 3 actuators (solid), ω_z with 2 actuators (dashed) [deg/s] vs time [s]; (c) $\alpha_{x,\mathrm{ref}}$ (dash–dot), α_x with 3 actuators (solid), α_x with 2 actuators (dashed) [deg/s] vs time [s]; (dashed) [deg/s] vs time [s]; (d) $\omega_{x,\mathrm{ref}}$ (dash–dot), ω_x with 3 actuators (solid), ω_x with 2 actuators (dashed) [m/s] vs time [s].

CONCLUSIONS

In this paper, we have analyzed a hybrid management of the control saturations, in the attitude control of a vehicle which considers also the roll dynamics. A hybrid control of three actuators allowed solving a tracking control problem and managing critical conditions, when multiple actuator saturations occur.

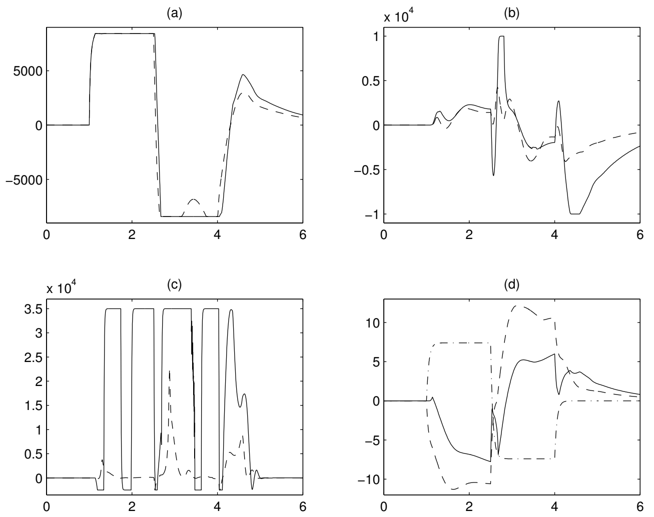

Fig. 4. (a) $F_{y,f}$ with the controller $\mathcal{C}_{sm}$ (solid), $F_{y,f}$ with the controller $\mathcal{C}_w$ (dashed) [N vs s]; (b) M_z with the controller $\mathcal{C}_{sm}$ (solid), M_z with the controller $\mathcal{C}_w$ (dashed) [Nm vs s]; (c) b_x with the controller $\mathcal{C}_{sm}$ (solid), b_x with the controller $\mathcal{C}_w$ (dashed) [N m rad/s vs s]; (d) δ_c with the controller $\mathcal{C}_{sm}$ (solid), δ_c with the controller $\mathcal{C}_w$ (dashed), δ_d (dash–dot) [deg vs s]

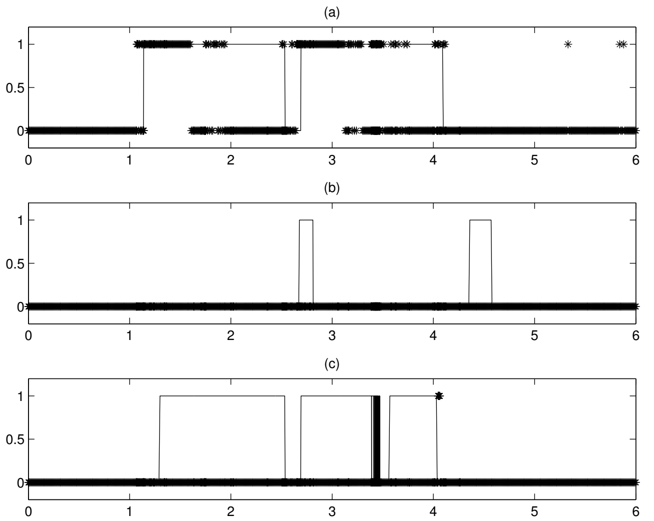

Fig. 5. Actuator saturation (1) or out of saturation (0) vs time [s], in the case of the controller $\mathcal{C}_{sm}$(solid) or with with the controller $\mathcal{C}_w$ (stars): (a) $\Delta F_{y,f}$; (b) M_z; (c) b_x

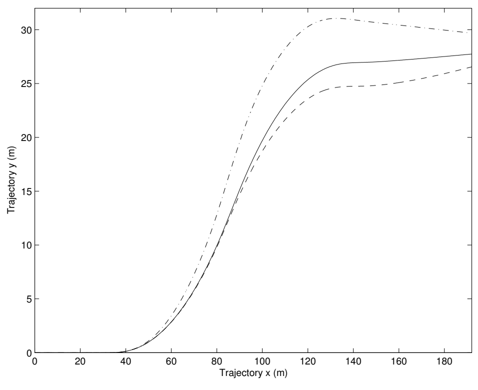

Fig. 6. Trajectory in the plane: reference (dash–dot), trajectory with the controller $\mathcal{C}_{sm}$ (solid), trajectory with the controller $\mathcal{C}_w$ (dashed)

Simulations results show the improvement of the proposed solution with respect to existing controllers and simpler saturation schemes.

This work suggests a number of future directions of research. From the theoretical point of view, the hybrid controller poses problems regarding the stability in presence of discontinuity of the control action. Additionally, the robustness of the proposed control strategy, taking into account model uncertainties/non–idealities, is an issue to be investigated. Finally, it would be interesting to extend the simulation setup to more complex simulation environments (e.g. Carsim), to show the robustness of the proposed approach in the presence of un–modeled dynamics.

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